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Keywords: axle load, maintenance, railways, infrastructure costs

JEL Codes: R48, L92

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Abstract

In this paper we estimate the impact of axle loads on rail infrastructure maintenance costs in Sweden. The results are contrasted to the cost impact of ton density, a common measure in the literature on rail infrastructure costs. We find non-linear cost elasticities with respect to a ton per axle density measure, with an estimate at the sample median that is larger than the corresponding estimate for ton density. The results are relevant for the setting of track access charges in Europe, considering that the econometric results in this paper give support to the engineering perspective - that is, axle loads are important to consider when estimating the damage caused by traffic.

1.0 Introduction

The vertical separation between train operations and infrastructure management in Europe during the 1990s made an introduction of track access charges necessary.[1] The charging principles, as set forth in 2001 by EU legislation (Dir. 2001/14), states that these charges should be set according to the direct cost of running a vehicle on the rail infrastructure. An important part of these charges concerns the wear and tear of the infrastructure, which will differ depending on the characteristics of the vehicle running on the tracks (as well as on the characteristics of the tracks). Differentiating the charge with respect to the vehicles' effect on the wear and tear can establish an efficient use of the infrastructure.

[1] The Swedish reform took place in 1988, preceding the wider European reform.

An important characteristic of the vehicle in this context is the axle load. Its effect on wear and tear became more apparent as the demand for improved efficiency in railway transportation required increased axle loads. Research was necessary in order to predict its effects and test whether an increased axle load was economically feasible or not. A source of knowledge on this matter are the studies in the Heavy Axle Load (HAL) Research Program, initiated in 1988 in the U.S., in which predictions on the cost impacts were made using engineering models for different components and bridges. The actual costs were however much lower than expected, partly due to better technology and maintenance management (Martland 2013). The International Union of Railways (UIC) also made a series of studies during the 1980s on increased axle loads, generating results that are utilized in for example Öberg et al. (2007). More specifically, the calculations in Öberg et al. show that an increase in the axle load from 16 to 22 tons will cause a 60 per cent increase in costs per ton-km. However, their approach relies on an assessment by experts within the Swedish Rail Administration; an assessment of the shares of maintenance and renewal costs that each deterioration mechanism has caused. This assessment may or may not be close to the actual cost shares of the different mechanisms, and the calculated cost impact from an increased axle load is therefore uncertain.

Both of the above-mentioned cases are so called bottom-up approaches, which are based on engineering models. In general, this type of approach predicts the damages caused by traffic and then links it to the cost of rectifying these damages, which requires assumptions on the amount of maintenance (and renewal) activities performed and their respective costs. Again, these assumptions may, or may not, generate predictions that are close to the actual costs caused by an increase in axle loads. There is thus reason to study the direct relationship between axle load and actual costs, which is the purpose of this paper.

A top-down (econometric) approach is used to estimate the cost elasticity of axle loads. More specifically, we estimate the cost impact of axle loads on rail infrastructure maintenance in Sweden using a panel dataset over the period 2011-2014. This may contribute to the knowledge of how different axle loads affect maintenance costs, and can be useful when setting track access charges for the wear and tear costs of the rail infrastructure.

Studies on the direct relationship between axle loads and maintenance costs are scarce. Nevertheless, according to Jansson (2002), the studies that have tried to establish estimates have found an almost linear or slightly increasing relationship.[2] The relationship between ton density and costs is for example analyzed in Johansson and Nilsson (2004), Andersson (2008), Wheat and Smith (2008), Gaudry and Quinet (2009), and Wheat et al. (2009). Still, these studies do not use (have access to) information on the axle loads of each train and therefore use a ton density measure; a measure that may hide information that is important for the wear and tear of the rail infrastructure and, hence, the maintenance costs. For example, a ton density measure does not reflect an increase in axle loads when a train operator moves a certain number of tons with one train instead of two trains (or one wagon instead of two wagons). Using this measure to estimate the cost impact of infrastructure usage may therefore produce biased results. For that reason, we make a comparison between a total ton measure and an axle load measure in this context, using econometric techniques.

The paper is organized as follows. Section 2 describes the estimation approaches and aspects that are important for estimating the cost impact of axle loads. The models we estimate are presented in section 3. Section 4 contains a description of our data. The estimation results are presented in section 5, while section 6 concludes.

[2] It is however not clear which studies Jansson (2002) is referring to.

2.0 Estimation approaches

We consider two estimation approaches to measure the cost impact of increased axle loads. In the first approach, we wish to estimate the cost impact of different axle load intervals. In the second approach, we consider a measure that includes the effect of increased axle loads.

The measures in the first approach (and its complications) is best described using a made up example of two identical track sections - that is, identical characteristics and production environments such as the same annual (and accumulated) tonnage. However, there is one aspect that differs: a large share of section *A*'s annual tonnage consists of trains with high axle loads, while the other section's (*B*) annual tonnage comprises more trains (and/or possibly more wagons per train). Track section *B* therefore has a larger share of its total ton density comprised by low axle loads. This is illustrated in Table 1 below.

One hypothesis is that section *A* will experience more wear and tear, which implies that the cost elasticity increases with ton per axle: $\beta_1 < \beta_2 < \beta_3$. That is, a proportional increase in ton density within the axle load interval 16 to 25 tons is expected to cause a higher proportional increase in maintenance costs compared to a corresponding proportional increase in ton density within the axle load interval 10 to 15 tons.

Table 1 – Axle load differences

Axle load interval, tons	Ton density* section A	Ton density* section B	Cost elasticity
1-9	100 000	1 500 000	β_1
10-15	1 100 000	1 000 000	β_2
16-25	2 500 000	1 200 000	β_3
Total ton density	3 700 000	3 700 000	

*Ton density = ton-km/route-km

On the other hand, the maintenance cost is not necessarily higher on section *A* compared to section *B*. One reason is that a high train density - creating the difference in axle loads between section *A* and *B* - implies shorter available time slots for maintenance and/or more maintenance during night-time, which is costly. To examine if this is the case in our dataset we calculate correlation coefficients for ton densities with respect to axle load intervals and train density. The correlation coefficients are estimated on traffic data during the period 2011-2014, which contains information on ton-km and the number of axles of each train. We have 707 observations in total. A further description of the data is provided in section 4.

The results are presented in Figure 1 below, which shows that the axle load intervals 9 to 11, 11 to 13, 13 to 15 and 15 to 17 is more correlated with train density compared to higher axle load intervals. These differences are statistically significant. For example, the difference between the correlation coefficients for the intervals 17 to 19 and 19 to 21 is statistically significant (z-score = 6.56, p-value = 0.000), where we use the method by Steiger (1980) to compare correlation coefficients. A table of z-scores for the differences between the correlation coefficients is presented in appendix.

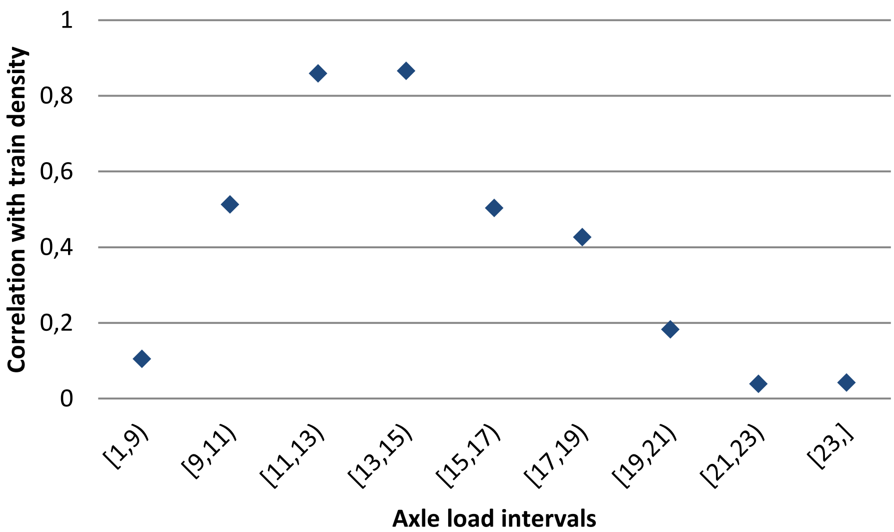

Figure 1 – Correlation coefficients for the sum of tons with respect to axle load intervals and train density in Sweden 2011-2014

Connecting the relationship illustrated in Figure 1 to track sections A and B in Table 1, it implies that section B will have a higher train density compared to section A.[3] This means that section A is costly to maintain due to high wear and tear from the axle loads, while section B will be costly to maintain due to its high train density. It may therefore be important to control for train density in order to identify the effect of different intervals of tons per axle.

The measure we consider in our second estimation approach is the sum of the trains' average tons per axle on each track section and year. Considering that all trains do not use the entire track section, we use a density measure.

$$\frac{\sum_{j=1}^{J} \frac{TKM_{jit}}{AXLES_{jit}}}{RL_{it}} \tag{1}$$

where $j = 1,2 \dots, J$ number of trains, $i = track\ section$ and $t = year$. TKM is ton-km and RL is route length in kilometers. This measure has an advantage over using tons for different intervals of axle loads as it requires fewer parameters to be estimated, which increases the efficiency of the estimator.

The two measures in our estimation approach reflect an increase in axle loads which is not revealed by a ton density measure; the most common traffic measure in the literature on rail infrastructure wear and tear costs.[4] For example, the axle load measures reflect the difference between two trains with the same total weight but with different number of axles. However, the measure in equation (1) does not reflect the difference between two trains with the same weight per axle but with a difference in total weight.

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[3] However, this will not be the case if the trains on section B have a lot of wagons (hence, many axles) that carries a similar amount of tons as the shorter trains on section A.

[4] The measure for ton density is $\frac{\sum_{j=1}^{J} TKM_{jit}}{RL_{it}}$.

To provide further intuition for the ton per axle density measure and its advantages (including its possible disadvantage with respect to total tons running on a section), we consider a train operator who runs one train between a certain origin and destination, but wishes to increase its shipment. The operator can do this by either a) increasing the load of each wagon; or b) use more wagons; or c) run two trains. An example of the options and the effects on the different traffic measures is shown in Table 2. The axle per ton density makes a distinction between options a) and b), indicating 20 and 10 tons per axle respectively, while the ton density measure increases with the same amount in both cases. Train density is unchanged in this case. Option c) is reflected by all of the three traffic measures.[5]

Table 2 – Comparison between measures of traffic

				Measures of traffic		
	Weight, tons	No. of axles	No. of trains	Ton per axle density (eq. 1)	Ton density	Train density
Starting Point	500	50	1	10	500	1
Options						
A	1000	50	1	20	1000	1
B	1000	100	1	10	1000	1
C	1000	100	2	20	1000	2

In order to take the total number of tons into account when option b) is chosen, we also need to consider the total ton density measure when estimating the effect of axle loads using the ton per axle density measure. Moreover, relating the ton per axle density measure to Figure 1, we note that it includes the effect of more trains running on a section. Hence, we cannot use this measure to *identify* the effect of increased axle loads. Rather, it is used to *include* the effect of increased axle loads, which the ton density measure is unable to do.

[5] Note that the ton per axle density is the sum of the trains' tons per axle, as specified in equation (1).

It should be noted that the axle load intervals and the measure in equation (1) do not *per se* treat an individual ton on a wagon with a total axle load at 25 tons differently from a ton on a wagon with a total axle load at 10 tons. The reason is that we do not have access to information on the weight of each wagon in the train set.[6]

Another aspect that may affect the estimation results is the differences in characteristics of the vehicles that contribute to wear and tear of the infrastructure, such as boogie type. Unfortunately, we do not have access to this information in this study.

A difference in network characteristics is yet another aspect that can be important to control for in the estimations. For example, increased axle loads require changes in the infrastructure to make it more resilient towards the deterioration caused by the higher forces from the vehicles. Investments in the rail infrastructure have been made on different parts of the railway network in Sweden, where the infrastructure manager (IM) is gradually increasing the maximum axle load allowed to 25 tons. Some parts of the network are designed for 30 tons per axle. These differences imply that the proportional increase in maintenance costs from a proportional increase in axle load on a train will be different depending on the maximum axle load allowed – that is, the cost elasticities will differ. Hence, controlling for this network characteristic is also important for identifying the effect axle loads has on maintenance costs.

Apart from the variables mentioned above, we use a number of control variables which we describe in the following two sections together with a presentation of the model we estimate.

[6] For example, two wagons with 25 and 10 tons per axle, respectively, have the same total ton per axle as two wagons with 24 and 11 tons per axle. However, the latter set of wagons is (slightly) better than the former from a wear and tear perspective as the eleventh ton per axle is less damaging than the 25th ton per axle.

3.0 Model

To estimate the effect of axle loads in our econometric (top-down) approach, we use a cost function given by equation (2), with $i = 1,2 \dots, N$ track sections observed over $t = 1, 2, 3, 4$ years.

$$C_{it} = f(\boldsymbol{X}_{it}, \boldsymbol{Q}_{it}, \boldsymbol{Z}_{it}), \tag{2}$$

C_{it} is maintenance costs, $\boldsymbol{X}_{it}$ is a vector of variables for infrastructure characteristics such as track length, rail age and maximum axle load allowed on the tracks. $\boldsymbol{Q}_{it}$ is a vector of traffic variables including ton density for different axle load intervals, tons per axle density, total ton density, and train density. $\boldsymbol{Z}_{it}$ is a vector of dummy variables for the five maintenance regions in Sweden (region East, West, South, North and Central) as well as year dummy variables to capture general effects over the rail network such as variations in input prices (however, we have access to the gross hourly wage for "building frame and related trade workers", which varies between different regions in Sweden as well as over time).

With access to panel data over the period 2011-2014, we are able to model unobserved track section specific (time-invariant) effects. If these effects are uncorrelated with the independent variables, we can use the random effects model. The parameter estimates will be biased if this assumption is not valid. The (less efficient) fixed effects model does not require this assumption to generate unbiased estimates. However, it relies on variation within groups (track sections) and is unable to produce (efficient) estimates of variables if their variation is time-invariant (low).

We use a double-log specification - that is, the dependent and independent variables are log-transformed - which is a useful transformation of data if the estimated residuals are skewed and/or we have problems with heteroscedasticity. We consider the following (Cobb-Douglas) model:

$$lnC_{it} = \alpha + \sum_{r=1}^{R} \beta_r lnX_{rit} + \sum_{k=1}^{K} \beta_k lnQ_{kit} + \sum_{m=1}^{M} \beta_m Z_{mit} + \mu_i + v_{it}, \qquad (3)$$

where α is a scalar, μ_i is the unobserved track section specific effects and v_{it} is the error term. β_r, β_k and β_m are parameters to be estimated for the R number of explanatory variables, K traffic variables and M dummy variables.

We also consider a Translog model, which is a second order approximation of a cost (production) function (see Christensen et al. 1971 and Christensen and Greene 1976). The Cobb-Douglas is nested in the Translog model, where the latter is more flexible and put few restrictions on the elasticities of production. For example, we allow for non-linear cost elasticities. Moreover, we consider cubic terms for the traffic variables in order to allow for turning points, such as the U-shaped curve that can indicate costs that decreases with output up to a certain point, and then increases with high enough output levels (diseconomies of scale).

4.0 Data

The rail network in Sweden is divided into five regional units, each administering a number of track sections for which we have data on costs, traffic and rail network characteristics. These regional units are called Region North, West, East, South and Central, for which we use dummy variables to capture management effects (as well as other regional effects that otherwise would end up as unobserved heterogeneity in the model estimation). Furthermore, it may be important to control for effects due to competitive tendering of maintenance production. Odolinski and Smith (2016) found an 11 per cent decrease in maintenance costs, using a dataset over the period 1999-2011. The reform was introduced in 2002, but has been gradual. This implies that there are a number of track sections during the period of our dataset that were not tendered in competition, and we therefore include a set of dummy variables to capture the effect of competitive tendering.

Information on maintenance costs has been retrieved from the Swedish Transport Administration, and includes costs from activities performed to maintain the assets of the railway network. These costs may vary depending on the technical aspects of the rail infrastructure. For example, the maximum axle load allowed can be an important characteristic to control for in order to isolate the cost impact of axle loads cost. We also have information on the average rail age, which can be a proxy for track standard due to the accumulated use of the tracks. Furthermore, we have information on the average quality class on a section which indicates the maximum speed allowed.

Descriptive statistics of the available data used in this study is presented in Table 3 and Table 4, where the latter covers the traffic variables. These observations are at the track section level during years 2011 to 2014. In total, we have access to 707 observations. Depending on the traffic variables we use in the model estimation, we lose a set of observations due to the log transformation as there are sections with zero values for certain axle load intervals during a year.

Table 3 – Costs, rail network characteristics and dummy variables, 2011-2014 (707 obs.)

	Median	Mean	St.dev	Min	Max
Maintenance costs, million SEK*	10.67	15.93	18.93	0.05	209.22
Hourly wage, SEK*	169.57	168.95	7.34	154.31	187.44
Route length, km	41.00	53.76	40.48	1.79	219.39
Track length, km	57.42	71.20	51.05	4.52	251.67
Average rail age	21.12	22.17	9.74	2.30	53.15
Average quality class (1 to 6)** (Qual_ave)	3.00	2.99	1.16	1.00	6.00
Switches, km	1.27	1.72	1.65	0.06	13.60
Length of structures (tunnels and bridges), km (structures_length)	0.42	1.59	3.63	0.01	22.08
Max.axle load allowed, tons	22.50	23.29	1.86	16.00	30.00
Year11	0	0.26	0.44	0	1
Year12	0	0.25	0.43	0	1
Year13	0	0.25	0.44	0	1
Year14	0	0.24	0.43	0	1
Region West	0	0.19	0.39	0	1
Region North	0	0.14	0.35	0	1
Region Central	0	0.18	0.38	0	1
Region South	0	0.26	0.44	0	1
Region East	0	0.24	0.43	0	1
Dummy for sections tendered in competition (CtendInd)	1	0.96	0.20	0	1
Dummy when tendered in competition (Ctend)	1	0.94	0.24	0	1
Dummy when mix between tendered and not tendered (Mixtend)	1	0.03	0.17	0	1

* 2014 prices, ** Track quality class ranges from 0-5 (from low to high line speed), but 1 has been added to avoid observations with value 0.

There are differences in tons per axle between different wagons on a train. We do not have access to this information. Instead we are left with the average ton per axle for each train. This information is used to create ton density variables for different axle load intervals as well as a ton per axle density variable. The intervals presented in Table 4 cover about two tons per axle each, apart from the first and last interval (1 to 5 and 25 and more tons respectively).

Table 4 – Traffic variables, 2011-2014 (707 obs.)*

	Median	Mean	St.dev	Min	Max
Train density	11.19	19.01	24.04	0.01	190.45
Ton density	4 405.02	7 866.47	9 173.37	0.42	65 793.97
Ton per axle density	177.95	300.71	398.60	0.06	5 304.19
Ton density [0, 5)	0.03	0.97	5.88	0.00	67.45
Ton density [5, 7)	27.40	180.12	505.60	0.00	5 374.56
Ton density [7, 9)	113.73	246.31	297.19	0.00	1 757.44
Ton density [9, 11)	204.26	501.92	669.50	0.00	4 616.28
Ton density [11, 13)	585.57	1 512.02	2 254.08	0.00	13 887.76
Ton density [13, 15)	880.21	2 522.59	5 328.65	0.00	47 310.77
Ton density [15, 17)	312.29	792.89	1 312.27	0.00	9 857.22
Ton density [17, 19)	127.55	336.66	527.39	0.00	3 896.97
Ton density [19, 21)	117.47	276.38	344.27	0.00	2 431.16
Ton density [21, 23)	73.31	395.62	614.78	0.00	2 765.98
Ton density [23, 25)	1.11	84.37	192.96	0.00	1 269.61
Ton density [25,)	292.79	1 016.61	2 650.90	0.00	30 947.37

***** The units of the traffic variables are expressed in thousands

5.0 Results

Three models are estimated. In the first model, we use all the axle load intervals available (*Model 1a*) and also test the inclusion of train density (*Model 1b*). The number of observations is 299 in Model 1 due to the zero values for the different traffic variables which are dropped when we use a log transformation of the data. The results are presented in appendix. To reduce the number of dropped observations, as well as reduce the number of parameters to be estimated, we create larger axle load intervals for the traffic variables in Model 2. Estimation results from this model are presented in Table 6.

In *Models 3a* and *3b* we use ton per axle density variable and ton density variable, respectively. *Model 3c* includes both traffic measures. These results are presented in Table 7.

Table 5 – Models estimated

Model	*Traffic variables*	*Results*
1a	Ton densities w.r.t. 12 intervals for tons per axle	Appendix, Table 9
1b	Ton densities w.r.t. 12 intervals for tons per axle and a train density variable	Appendix, Table 9
2a	Ton densities w.r.t. 5 intervals for tons per axle	Table 6
2b	Ton densities w.r.t. 5 intervals for tons per axle and a train density variable	Table 6
3a	Ton per axle density: equation (1)	Table 7
3b	Ton density	Table 7
3c	Ton density and ton per axle density: equation (1)	Table 7

Before elaborating on the specific results in each model, we note that the wage variable turned out to produce estimates against economic theory (negative coefficient, yet non-significant). Hence, we exclude this input price variable in our preferred models and rely on the region dummies to capture geographical differences in wages. Their time-varying effect will be captured by the year dummies.

In general, the traffic variables in our models are rather slow-moving (*sluggish*), making the fixed effects estimator unhelpful in providing useful estimates. We therefore present the estimation results from the random effects estimator even though these may be biased due to a correlation between the unobserved track section specific effects and the regressors (the Hausman test suggests that we use the fixed effects estimator). Still, it should be noted that we control for a large set of infrastructure characteristics at the track section level, as well as time-invariant effects that relate to the regional unit the sections belong to. Moreover, the estimation results in Model 3 are compared with previous estimates on Swedish data on a longer time period (1999-2014), where the fixed effects estimator is used. These results are similar.

Given that the full dataset can be used in Model 3, which also generates plausible results for the traffic variables, we only comment briefly on the traffic variables in models 1-2 and focus on the complete estimation results from Model 3 in section 5.2, which is our preferred model. All estimations are carried out using Stata 12 (StataCorp.2011).

5.1 Estimation results, Model 1 and Model 2

We first note that Model 1 generates rather strange results. The coefficient for the axle load interval [17, 19) is negative and statistically significant, while the coefficients for some of the lower axle load intervals are positive and statistically significant. In fact, axle load intervals [11, 13), [13, 15) and [15, 17) have the highest estimates; intervals that are also highly correlated with train density (see Figure 1). Indeed, these estimates become statistically insignificant when we include the train density variable (*Model 1b*). See Table 9 in appendix.

The results in Model 1 may indicate that the 299 observations are too few (with too low variation) for the purpose of this study. Model 2 includes more observations as the larger intervals for the axle load variables create fewer observations with zero values. However, even with 665 observations, we get similar results as in Model 1. The coefficient for the axle load interval [9, 15) is statistically significant and larger than the estimates for the other intervals, but becomes statistically insignificant and slightly negative when we include train density in the estimations (*Model 2b*).

Table 6 – Estimation results, Model 2 (random effects)

	Model 2a		*Model 2b*	
	Coef.	Rob. std. Err.	Coef.	Rob. std. Err.
Cons.	16.3803***	0.2316	16.4884***	0.1866
Train density	-	-	0.3134***	0.0680
Ton density [0,9)	0.0137	0.0370	0.0274	0.0361
Ton density [9,15)	0.1105***	0.0336	-0.0055	0.0335
Ton density [15,19)	0.0249	0.0230	0.0072	0.0242
Ton density [19,23)	-0.0438	0.0334	-0.0432	0.0318
Ton density [23,)	0.0052	0.0198	-0.0157	0.0179
Track_length	0.6316***	0.0768	0.6445***	0.0690
Rail_age	0.1034	0.0806	0.0907	0.0748

Qual_ave	0.2810**	0.1136	0.4185***	0.1045
Switch_length	0.2259***	0.0564	0.1950***	0.0519
Structures_length	0.0745*	0.0423	0.0473	0.0376
Max.axle_load_allowed	0.8781*	0.5141	0.7992	0.5226
Year12	0.1365***	0.0429	0.1202***	0.0431
Year13	0.2511***	0.0475	0.2405***	0.0476
Year14	0.4183***	0.0594	0.3958***	0.0573
Region North	-0.0250	0.1936	0.1425	0.1717
Region Central	-0.2743**	0.1326	-0.2157*	0.1179
Region South	-0.3515***	0.1165	-0.3889***	0.0964
Region East	-0.2247*	0.1284	-0.2645**	0.1076
Ctend	-0.5630***	0.1691	-0.5300***	0.1624
Mixtend	-0.2655**	0.1209	-0.2177*	0.1126
CtendInd	0.4138*	0.2397	0.2528	0.2039
No. Obs.	665		665	
Mean VIF	3.40		3.65	

Note: ***, **, *: Significance at the 1%, 5%, 10% level.

5.2 Estimation results, Model 3

In Model 3, we use the ton per axle density variable and compare the results with the ton density variable. The coefficients for ton per axle density are significant and indicate that the cost elasticity with respect to tons per axle varies for different traffic levels, as well as with different levels of maximum axle loads allowed. The first order coefficient (which is evaluated at the sample median) is 0.2377 (p-value = 0.000).

A non-linear relationship and an interaction with the maximum axle load allowed could not be retained in the translog model using ton density as the output variable (*Model 3b*; Chi2=3.36, Prob>chi2=0.305). The cost elasticity with respect to ton density is 0.1765 (p-value=0.000), which is very close to the results in Odolinski and Nilsson (2015), who use the fixed effects estimator on Swedish data during the period 1999-2014. Their estimate is 0.1719 at the sample mean. In general, our estimates are rather similar to their study. However, our estimates may still be somewhat biased due to the random effects estimator. Nevertheless, we consider the comparison between the traffic variables in our model estimations rather robust, as the ton density estimate is plausible and close to previous (unbiased) estimates on Swedish data.

We note that the coefficient for maximum axle load allowed is negative at the sample median, indicating that these tracks are less costly to maintain when controlling for other factors. This is what one can expect given that these tracks are designed to withstand higher loads. This view is corroborated by the estimate for the interaction term between maximum axle load and tons per axle density. Specifically, the coefficient is negative and statistically significant, which indicates that the cost impact of axle loads is lower when the tracks are designed to be more resilient against heavy loads.

The estimation results in *Models 3a* and *3b* show that the cost elasticity for ton per axle density is higher that the corresponding estimate for ton density (0.2377 and 0.1765, respectively). However, as shown in Table 2, the ton per axle measure does not fully capture the total amount of tons that has been running on a track section. We therefore include the ton density measure in *Model 3c*, being aware of the rather high correlation coefficient between the logarithm of these traffic variables (0.8954). The correlation between these variables is reflected by the estimation results in *Model 3c*, in which the estimate for ton per axle ton density drops about 0.04 percentage points, similar to the estimated effect of ton density (0.0511). Comparing the estimates for the traffic measures in *Models 3a-c*, the estimate for ton per axle density in *Model 3a* seems to (partly) capture the effect of ton density.

Table 7 - Estimation results, model 3 (random effects)

	Model 3a		*Model 3b*		*Model 3c*	
	Coef.	Rob. std. err.	Coef.	Rob. std. err.	Coef.	Rob. std. err.
Cons.	16.5536***	0.2497	16.4261***	0.2411	16.5577***	0.2471
Ton_den.	-	-	0.1765***	0.0352	0.0511	0.0439
Ton_per_axle_den.	0.2377***	0.0402	-	-	0.1926***	0.0585
Ton_per_axle_den.^2	0.0489	0.0343	-	-	0.0451	0.0328
Ton_per_axle_dens.^3	0.0186	0.0189	-	-	0.0172	0.0177
Max.axle_load_allowed	-1.5134*	0.8632	-0.7572	0.7296	-1.5761*	0.8573
Track_length	0.5683***	0.0654	0.5466***	0.0693	0.5626***	0.0656
Rail_age	0.1382**	0.0696	0.1171*	0.0695	0.1379**	0.0691
Qual_ave	0.3489***	0.0962	0.2799***	0.0974	0.3570***	0.0961
Switch_length	0.1830***	0.0497	0.2126***	0.0510	0.1797***	0.0500
Structures_length	0.1083***	0.0359	0.1161***	0.0419	0.1101***	0.0362
Year12	0.1252***	0.0394	0.1243***	0.0381	0.1229***	0.0387
Year13	0.1974***	0.0485	0.2552***	0.0489	0.2078***	0.0492
Year14	0.3366***	0.0513	0.4351***	0.0494	0.3539***	0.0540
Region North	-0.2123	0.1786	-0.2383	0.1691	-0.2327	0.1768
Region Central	-0.2781**	0.1181	-0.3039**	0.1375	-0.2936**	0.1201
Region South	-0.4351***	0.1072	-0.3518***	0.1106	-0.4340***	0.1077
Region East	-0.3501***	0.1165	-0.2684**	0.1178	-0.3564***	0.1162
Ton.axle_den.Max.axle_l.	-0.7695**	0.3247	-	-	-0.7332**	0.3361
Max.axle_load_allowed^2	26.2970***	8.0867	17.6814***	6.7808	25.6902***	8.2035
Mixtend	-0.1654	0.1345	-0.2511*	0.1368	-0.1676	0.1370
Ctend	-0.4146**	0.1680	-0.4735***	0.1730	-0.4122**	0.1702
CtendInd	0.1210	0.2259	0.2586	0.2407	0.1226	0.2286
No. Obs	707		707		707	
Mean VIF	4.02		3.11		4.62	

The variables were divided with the sample median prior to taking logs. The first order coefficients can therefore be interpreted as cost elasticities at the sample median.

Note: ***, **, *: Significance at the 1%, 5%, 10% level.

The coefficients for track length, rail age, length of switches, and track lengths of structures (tunnels and bridges) have the expected signs and are statistically significant. Average quality class is also positive and significant, indicating that lower line speeds that are associated with less strict track geometry requirements are more costly to maintain.

We find that Region West - which is the baseline in the model - has higher maintenance costs compared to the other regions, where Region South has the lowest estimate. Furthermore, the coefficient for competitive tendering (Ctend) is negative in all model estimations, where we also include a time-invariant dummy variable that indicate which track sections that are tendered at least a full year during 2011-2014. This is in line with a difference-in-differences approach (see for example Greene 2012, pp. 155-7). Note that we do not have a general post-competitive tendering period (in that case we would include a dummy variable for these years). Instead, the year dummy variables control for the general time effects that are not to be confounded with the effects of competitive tendering.

The estimation results in Table 7 indicate a non-linear relationship between tons per axle density and maintenance costs. To illustrate this relationship, we estimate the cost elasticity ($\hat{\gamma}_{it}$) at the observed levels of output (Q_{it}), using the following equation:

$$\hat{\gamma}_{it} = \hat{\beta}_1 + 2 \cdot \hat{\beta}_2 \ln Q_{it} + 3 \cdot \hat{\beta}_3 (\ln Q_{it})^2, \quad (4)$$

where $\hat{\beta}_1$ is the first order coefficient for output, while $\hat{\beta}_2$ and $\hat{\beta}_3$ are coefficients for the squared and cubic terms of output, respectively. The non-linear relationship between ton per axle density and maintenance costs from *Model 3a* is illustrated in Figure 2 below, which includes a (normal-based) 95 per cent confidence interval based on standard errors estimated with the bootstrap method using 100 replications for each traffic volume in the sample (see for example Mooney and Duval 1993 for a thorough treatment of the bootstrapping method). As can be seen in equation (4), we exclude the interaction with maximum axle load allowed in this calculation, as this variable generates similar curves, yet at different levels. Figure 2 shows a U-shaped relationship between the cost elasticities and levels of traffic (as measured by tons per axle density). The cost impact is quite high for track sections with low levels of traffic, and drops dramatically when traffic increases. However, a turning point comes quickly, indicating higher cost elasticities for higher levels of traffic, yet at a decreasing rate.

Five observations have cost elasticities above 0.8 (which are not included in Figure 2 below). We re-estimated our model excluding these observations, which produced similar results.

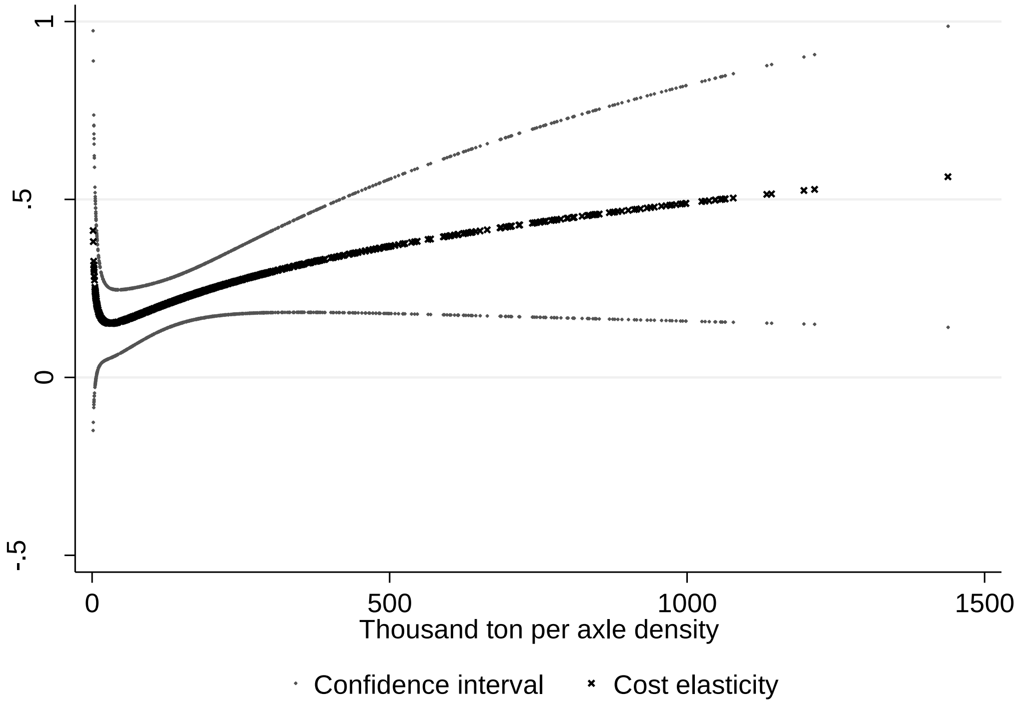

Figure 2 – Cost elasticity for different levels of ton density per axle, with 95 per cent confidence intervals

6.0 Conclusion

A direct relationship between axle loads and maintenance costs has been estimated in this paper, using a top-down (econometric) approach on Swedish data during the period 2011-2014. The estimation results indicate an increasing relationship between tons per axle density and maintenance costs – that is, the proportional increase in maintenance costs, due to a proportional increase in tons per axle density, is increasing with traffic levels (as measured by tons per axle density). The corresponding estimate for ton density is somewhat lower and turns insignificant (and decreases) when included along the tons per axle variable.

Given that the ton per axle measure gives a better representation of the wear and tear of the rail infrastructure compared to only using a ton density measure, the estimates in this paper can be informative for a vertically separated railway system where track access charges are required. The results are an indication of the importance of axle loads (as well as train density) with respect to maintenance costs, and more data can prove to be fruitful in the future. First, more observations can provide more robust results. Second, more detailed data can also be rewarding. There are for example differences in tons per axle between wagons on each train, which is information that was not available in this study. Instead, we are left with the average ton per axle for each train. Moreover, we did not have access to differences in characteristics between vehicles such as boogie type (which may be correlated with certain axle loads), which can also affect the results.

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Appendix

Table 8 – Z-scores for differences between correlation coefficients (std. err. in parentheses)

	[1,9)	[9,11)	[11,13)	[13,15)	[15,17)	[17,19)	[19,21)	[21,23)
[9,11)	-11.49***							
	(0.0402)							
[11,13)	-23.64***	-16.91***						
	(0.0501)	(0.0427)						
[13,15)	-22.70***	-15.81***	-0.64					
	(0.0533)	(0.0474)	(0.0436)					
[15,17)	-8.85***	0.26	17.02***	14.39***				
	(0.0507)	(0.0491)	(0.0431)	(0.0530)				
[17,19)	-7.52***	2.56***	17.63***	17.77***	2.01**			
	(0.0466)	(0.0435)	(0.0473)	(0.0485)	(0.0491)			
[19,21)	-2.23**	11.50***	24.76***	21.28***	7.70***	6.56***		
	(0.0358)	(0.0332)	(0.0446)	(0.0532)	(0.0480)	(0.0412)		
[21,23)	1.83*	13.00***	24.66***	23.71***	10.29***	8.81***	5.23***	
	(0.0364)	(0.0406)	(0.0507)	(0.0539)	(0.0501)	(0.0473)	(0.0280)	
[23,)	2.64***	10.49***	24.16***	23.93***	9.72***	7.95***	3.03***	-0.07
	(0.0238)	(0.0500)	(0.0516)	(0.0532)	(0.0527)	(0.0520)	(0.0471)	(0.0488)

Note: ***, **, *: Significance at the 1%, 5%, 10% level.

Table 9 – Estimation results, model 1 (random effects)

	Model 1a		*Model 1b*	
	Coef.	Rob. std. Err.	Coef.	Rob. std. Err.
Cons.	16.6286***	0.1757	16.7013***	0.1597
Train density	-	-	0.2731***	0.0842
Ton density [0,5)	0.0275	0.0188	0.0323*	0.0185
Ton density [5,7)	0.0023	0.0190	0.0087	0.0175
Ton density [7,9)	-0.0154	0.0424	-0.0254	0.0395
Ton density [9,11)	-0.0281	0.0383	-0.0212	0.0371
Ton density [11,13)	0.0613*	0.0350	0.0327	0.0345
Ton density [13,15)	0.1412***	0.0349	0.0429	0.0391
Ton density [15,17)	0.0667**	0.0306	0.0267	0.0291
Ton density [17,19)	-0.0795***	0.0306	-0.0737**	0.0305
Ton density [19,21)	-0.0361	0.0327	-0.0116	0.0345
Ton density [21,23)	0.0048	0.0264	-0.0008	0.0252
Ton density [23,25)	0.0130	0.0164	0.0112	0.0161
Ton density [25,)	-0.0090	0.0207	-0.0067	0.0190
Track_length	0.5358***	0.0871	0.5696***	0.0954
Rail_age	0.2061***	0.0703	0.1926***	0.0646
Qual_ave	0.2594**	0.1079	0.3492***	0.1043
Switch_length	0.2133***	0.0485	0.1746***	0.0544
Structures_length	0.0949*	0.0524	0.0783	0.0491
Max.axle_load_allowed	0.8458	0.5495	0.8290	0.5675
Year12	0.0024	0.0559	0.0052	0.0537
Year13	0.1963***	0.0659	0.1969***	0.0630
Year14	0.4036***	0.0706	0.4268***	0.0699
Region North	0.0342	0.1463	0.1362	0.1539
Region Central	-0.3053**	0.1457	-0.2809*	0.1485
Region South	-0.3163***	0.1222	-0.3335***	0.1131
Region East	-0.0985	0.1296	-0.1392	0.1196
Ctend	-0.1078	0.2433	-0.1394	0.2376
Mixtend	-0.0111	0.1711	-0.0300	0.1591
CtendInd	-0.1779	0.2260	-0.2539	0.2230
No. Obs.	299		299	
Mean VIF	4.19		4.46	

Note: ***, **, *: Significance at the 1%, 5%, 10% level.